Lagrangean based lower bounds for a multi-plant lot-sizing problem with capacity constraints

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The paper addresses a multi-item, multi-plant lot-sizing problem with capacity restrictions. A set of facilities (plants) is available for producing some items. For each period of a discrete planning horizon, a demand is defined for each pair of item and plant. The problem consists in producing all the demands such that the total production, inventory, setup and transfer costs is minimized. Setup production times are considered as well as capacity constraints on the production. Moreover, transfers between plants are allowed, however, the total transferred quantity between each pair of plants is upper bounded as well as the total inventory at each plant for a given period. The problem considered is NP-hard. We quote the work of Sambivasan and Yahya [1] that describes some Lagrangean-based heuristics to solve a relaxed version of the problem where no transfer and storage capacities are considered. In the present work, we propose a Lagrangean lower bound on the optimal cost value of the problem based on the decomposition of the problem into Facility Location and Multi-Commodity Flow problems.

Two formulations of the problem are proposed. An aggregate mixed integer formulation and a multi-commodity flow model where fixed costs are considered for the production arcs. Capacity constraints and fixed cost and time parameters are the main difficulties encountered when solving the problem (refer to [2]). Our work describes two decompositions and each reduces one of these two difficulties. We decompose the problem into a master facility location problem through Lagrangean relaxation of the capacity constraints, and a slave problem which reduces to a minimum cost multi-commodity flow problem if we consider the linear relaxation of the problem formulation.

The rationale of the algorithm is the following. A subset of producers (plants) for which the fixed times and costs are considered is determined when solving the master problem. The corresponding variables are fixed in the second phase when solving the slave problem which consists in a minimal cost multi-commodity flow without fixed cost and time parameters.

The solving method leads to the computation of a lower bound on the optimal objective value of the problem. The obtained solution may not be feasible. The process is thus repeated by considering the non-satisfied demands and the residual capacities until the stopping criteria is reached. Computational results showing the efficiency of the proposed lower bound will be reported.

Keywords: Lot-sizing, Capacity constraints, Facility location, Multi-commodity flows, Lagrangean relaxation

References

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